

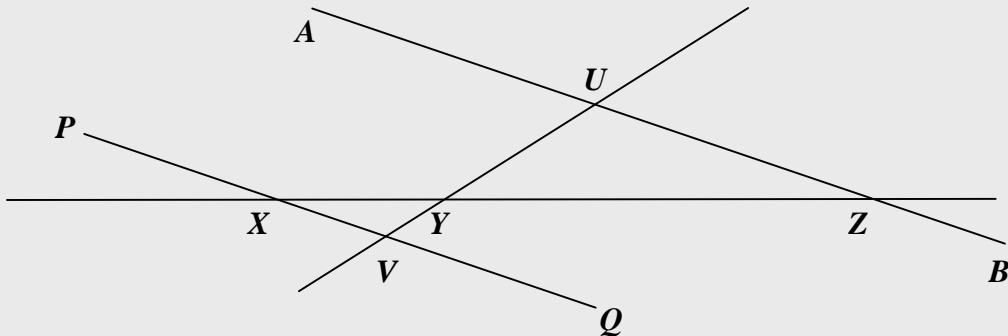
Examples 5 in Basic Geometry

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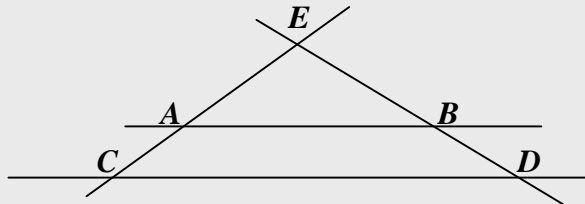
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Examples 5

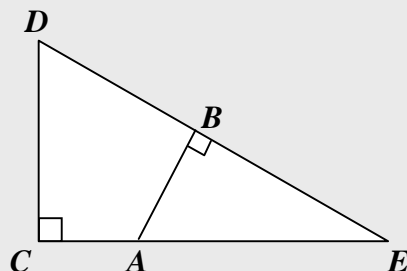
0. Assuming AB is parallel to PQ , $XY = 5$, $VY = 3$, and $YZ = 12$, find UY .



1. Assuming AB is parallel to CD , $AC = 2$, and $AE = 3$, find the ratio between AB and CD , and the ratio between BE and BD .



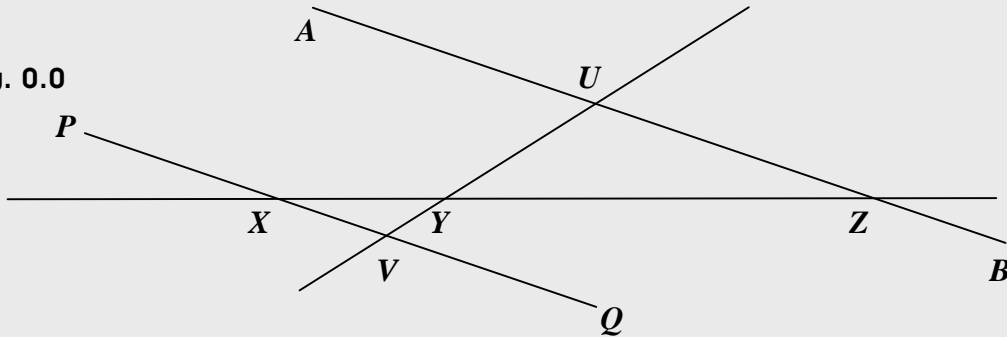
2. Assuming $AE = 5$, $AB = 3$, and $AC = 2$, find the lengths of BE , BD , and CD , and the ratio between the area of the triangle CDE and the area of the triangle BAE .



Suggestions or Solutions To the Problem 0

Assuming in the figure below, AB is parallel to PQ , $XY = 5$, $VY = 3$, and $YZ = 12$, find UY .

Fig. 0.0

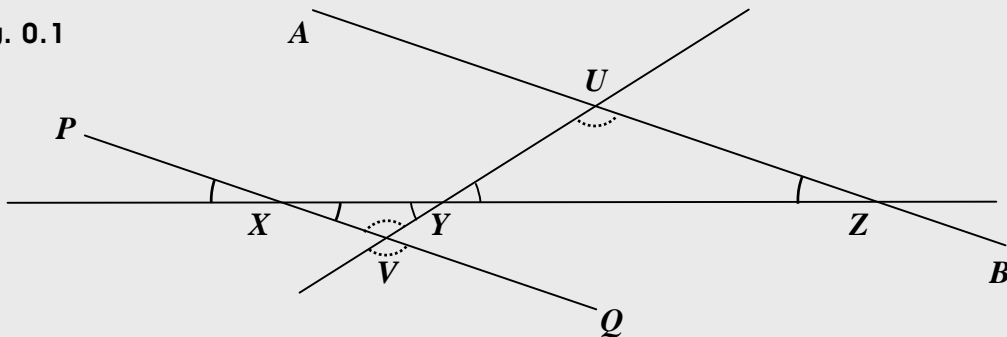


To begin with, we can see two triangles, one is $\triangle XVY$, and the other is $\triangle ZYU$.

And we can say that the two are similar triangles. Why?

Since the two lines AB and PQ are parallel, the two triangles have the same set of three angles, and we can specify all the angles in the two triangles the way below.

Fig. 0.1



And if two triangles are similar, each pair of corresponding sides keeps the same ratio.

In this case, XY corresponds to ZY , VY corresponds to UY , and VX corresponds to UZ .

So we get $XY/ZY = VY/UY = VX/UZ$. How then can we get the length of UY ?

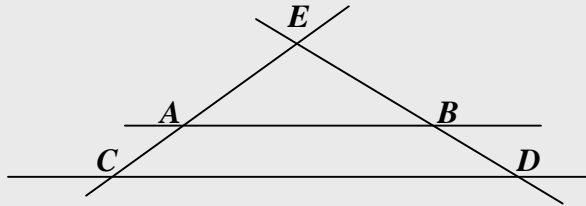
We have $XY = 5$, $VY = 3$, and $YZ = 12$, which is ZY , too, of course.

$$\text{So we get } \frac{XY}{YZ} = \frac{5}{12} = \frac{VY}{UY} = \frac{3}{UY} \Rightarrow \frac{5}{12} = \frac{3}{UY} \Rightarrow UY = \frac{3 \cdot 12}{5} = \frac{36}{5}.$$

Suggestions or Solutions To the Problem 1

Assuming in the figure below, AB is parallel to CD , and $AC = 2$, $AE = 3$, find the ratio between AB and CD , and the ratio between BE and BD .

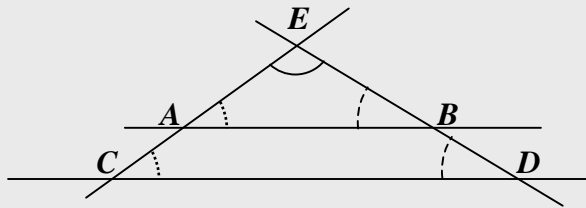
Fig. 1.0



To begin with, we can see two triangles, one is $\triangle ABE$, and the other is $\triangle CDE$. And we can say that the two are similar triangles. Why?

Since the two lines AB and CD are parallel, the two triangles have the same set of three angles, and we can specify all the angles in the two triangles the way as follows.

Fig. 1.1



And if two triangles are similar, each pair of corresponding sides keeps the same ratio. In this case, EA corresponds to EC , AB corresponds to CD , and EB corresponds to ED .

So we get $\frac{EA}{EC} = \frac{AB}{CD} = \frac{EB}{ED}$.

How then can we get the ratio between BE and BD ?

To begin with, we have $AC = 2$ and $AE = 3$, which is EA , too, of course.

So we get $EA + AC = EC = 5$. Thus, we get $\frac{EA}{EC} = \frac{AB}{CD} = \frac{EB}{ED} = \frac{3}{5}$.

What then, about the ratio of EB to BD , that is, what is $\frac{EB}{BD}$?

Assuming for instance, $EB = 6$, we get $\frac{EB}{ED} = \frac{3}{5} \Rightarrow ED = EB \cdot \frac{5}{3} = 10$.

Then, the difference between ED and EB is 4, which is BD .

Then, we get $\frac{EB}{BD} = \frac{6}{4} = \frac{3}{2}$.

Assuming next, $EB = 18$, we get $ED = 30$, because $\frac{EB}{ED} = \frac{3}{5}$.

Then, the difference between ED and EB is 12, which is BD , and we get

$$\frac{EB}{BD} = \frac{18}{12} = \frac{3}{2}.$$

How does though, the ratio not change, that is, $\frac{EB}{BD}$ is still $\frac{3}{2}$?

That's because of the distributive law.

For instance, applying the distributive law to subtractions, we can get $3p - 3q = 3(p - q)$.

Then, we can see that $3p : 3(p - q) = p : (p - q)$, and that $3q : 3(p - q) = q : (p - q)$.

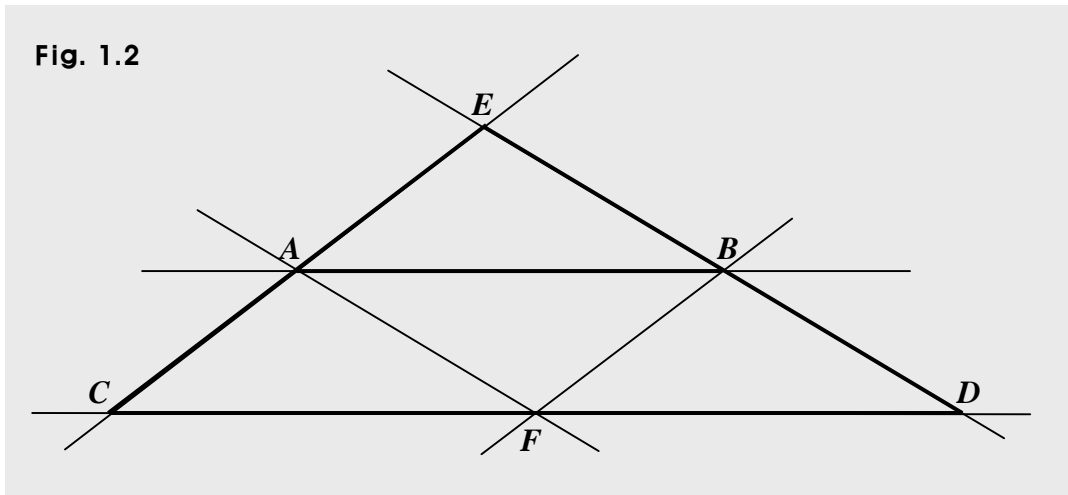
Thus, we can say that the ratio of EB to BD is $\frac{3}{2}$, that is, $\frac{EB}{BD} = \frac{3}{2}$.

What then about the ratio, $\frac{EA}{AC}$?

It is the same as $\frac{EB}{BD}$, which is $\frac{3}{2}$. So we get $\frac{EA}{AC} = \frac{3}{2}$, too.

That is to say that $\frac{EA}{AC} = \frac{EB}{BD} = \frac{3}{2}$.

Thus, suppose in the figure below, $AB \parallel CD$, that is, AB is parallel to CD , $EC \parallel BF$, and $ED \parallel AF$.



Then, we get these:

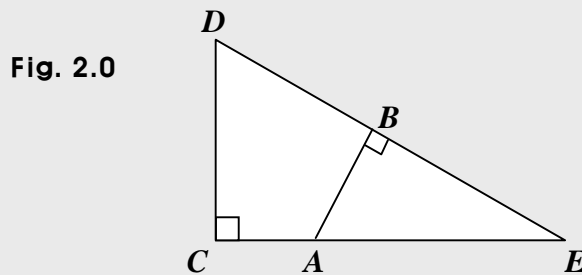
$$\frac{EA}{EC} = \frac{AB}{CD} = \frac{EB}{ED}.$$

$$\frac{EA}{AC} = \frac{CF}{FD} = \frac{EB}{BD}.$$

$$\frac{EC}{AC} = \frac{CD}{FD} = \frac{ED}{BD}.$$

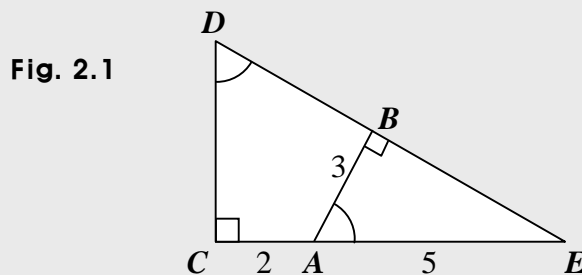
Suggestions or Solutions To the Problem 2

Assuming in the figure below, $AE = 5$, $AB = 3$, and $AC = 2$, find the lengths of BE , BD , and CD , and the ratio between the area of $\triangle CDE$ and the area of $\triangle BAE$.



First off, we can see two triangles similar, one is $\triangle CDE$, and the other is $\triangle BAE$.

So we can put the figure above more specifically the way below.



And if two triangles are similar, each pair of corresponding sides keeps the same ratio.

In this case, AE corresponds to DE , AB corresponds to DC , and BE corresponds to CE .

So we get $\frac{AE}{DE} = \frac{AB}{DC} = \frac{BE}{CE}$.

Then first, how can we get BE , BD , and CD ?

To begin with, $\triangle BAE$ is a right triangle, BE is one of the two legs in the right triangle. And we know that the other leg is 3, and the hypotenuse is 5.

So assuming $BE = b$, and using the distance formula, we get this:

$$5^2 = 3^2 + b^2 \Rightarrow b^2 = 25 - 9 = 16 = 4^2 \Rightarrow b = \pm 4.$$

And we know $b = BE > 0$. So we get $BE = 4$.

And let's next, move on to CD .

Then, finding it, we can use $CE = 7$, and a ratio, which is $\frac{AB}{BE}$, which is $\frac{3}{4}$.

That's because $\triangle CDE$ is similar to $\triangle BAE$. So we get $\frac{DC}{CE} = \frac{AB}{BE}$. How?

We have $\frac{AB}{DC} = \frac{BE}{CE}$.

So multiplying both sides by DC , we get

$$AB = DC \cdot \frac{BE}{CE}. \text{ Next, dividing both sides by } BE, \text{ we get } \frac{AB}{BE} = \frac{DC}{CE}.$$

Now, we know $\frac{AB}{BE} = \frac{3}{4}$, $\frac{AB}{BE} = \frac{DC}{CE}$, and $CE = 7$.

$$\text{So we get } \frac{DC}{CE} = \frac{DC}{7} = \frac{3}{4} \Rightarrow DC = \frac{21}{4}.$$

What then, about BD ?

We can put it this way $BD = DE - BE = DE - 4$. So we get $DE = BD + 4$.

And we know $\triangle CDE$ is a right triangle, DE is the hypotenuse, and the two legs are as follows. $DC = \frac{21}{4}$, and $CE = 7$.

So using the distance formula, we can get DE , and in turn, can get BD .

This time, too, though, we may want to use a ratio $\frac{AB}{AE}$, which is $\frac{3}{5}$.

That's because $\triangle CDE$ is similar to $\triangle BAE$. So we get $\frac{AB}{AE} = \frac{DC}{DE}$. How?

We have $\frac{AE}{DE} = \frac{AB}{DC}$. So multiplying both sides by DC , we get

$$AB = DC \cdot \frac{AE}{DE}. \quad \text{Next, dividing both sides by } AE, \text{ we get } \frac{AB}{AE} = \frac{DC}{DE}.$$

Now, we know $\frac{AB}{AE} = \frac{3}{5}$, $\frac{AB}{AE} = \frac{DC}{DE}$, and $DC = \frac{21}{4}$.

$$\text{So we get } \frac{DC}{DE} = \frac{3}{5} \Rightarrow DE = \frac{21}{4} = \frac{5}{3} = \frac{35}{4}.$$

$$\text{And we know } BD = DE - 4. \text{ So we get } BD = \frac{35}{4} - 4 = \frac{(35-16)}{4} = \frac{19}{4}.$$

And let's next, move on to the ratio between the area of $\triangle CDE$ and the area of $\triangle BAE$. How then can we get the ratio?

We can get the ratio, of course, if we find the areas of the two triangles.

We know however, that the two triangles are similar. So using the ratio between the corresponding sides, we can get the ratio between the two areas.

What then is the ratio?

It is the square of the ratio between the corresponding sides. What then, is the ratio between the corresponding sides?

We have $\frac{AE}{DE} = \frac{AB}{DC} = \frac{BE}{CE}$. And we know $BE = 4$, and $CE = 7$.

And we know that BE belongs to $\triangle BAE$, and CE belongs to $\triangle CDE$.

So the ratio of the area of $\triangle BAE$ to the area of $\triangle CDE$ is $\frac{4^2}{7^2}$, which is $\frac{16}{49}$.